Unit 5: Systems of Linear Equations and Inequalities

Time Frame: Quarter 2

Connections to Previous Learning:

Students solve linear equation in one or two variables, applying graphical and algebraic methods to solve systems of linear equations, and analyze solutions in context. Students build upon previous understandings of linear equations and functions and apply them to various representations of linear relationships, including tables, graphs, and equations. Students synthesize and apply their knowledge of linear equations and inequalities to model data. By-hand graphing skills will quickly be supplemented by technology, allowing for inspection and discussion of several data sets. Students use their knowledge of expressions and equations to evaluate functions and to rewrite when necessary. They also use rigid transformations and coordinate geometry as examples of functions and relations.

Focus within the Grade Level:

Students generate and interpret linear equations and inequalities to model data in real-world contexts. Students interpret key features, such as intercepts, slope, correlation, causation, and linear fit. Students become fluent in writing equations, interpreting variables, and verifying solutions as viable in the context of a problem. They explore systems of equations and inequalities, find and interpret solutions. Students understand that functions have exactly one output for every input and that functions can be expressed and described in multiple ways. Students become fluent with function notation and will be able to evaluate functions at various inputs and outputs. Students explore a variety of functions and representations and see function examples from algebra and real world experiences.

Connections to Subsequent Learning:

Students need a solid understanding of functions as they use different functions to model new phenomena and investigate particular function types. Later units will call on students’ understanding of modeling and various functions, including linear, exponential, piecewise, and absolute value to compare and choose appropriate models. In future work, students create and solve equations, inequalities, and systems of equations involving exponential and quadratic functions.
Common Core Standards in this unit:

**Solve systems of equations**

REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
REI.6 Solve systems of linear equations and approximately (e.g., with graphs) focusing on pairs of linear equations in two variables.

**Represent and solve equations and inequalities graphically.**

REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
REI.11 Explain why the x-coordinates of the points where the graphs of the equation \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear.
REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

*Estimated Days on the Unit 15 - 20 days*

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sure of structure.
8. Look for and express regularity in repeated reasoning.
### 8th Grade ALDs

**RANGE ALD**  
Target: Analyze and solve linear equations and pairs of simultaneous linear equations.

| Level 1 students should be able to solve linear equations in one variable with integer coefficients. | Level 2 students should be able to analyze and solve systems of linear equations graphically by understanding that the solution of a system of linear equations in two variables corresponds to the point of intersection on a plane. They should be able to solve and produce examples of linear equations in one variable with rational coefficients with one solution, infinitely many solutions, or no solution. | Level 3 students should be able to classify systems of linear equations as intersecting, coinciding, or parallel. Solve linear systems algebraically and estimate solutions using a variety of approaches; and show that a particular linear equation has one solution, no solution, or infinitely many solutions by successively transforming the given equation into simpler forms until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers). They should be able to solve and produce examples of linear equations in one variable, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. | Level 4 students should be able to analyze and solve problems leading to two linear equations in two variables in multiple representations. |

**THRESHOLD ALD**  
Expressions and Equations  
Targets B, C, and D

| The student who just enters Level 2 should be able to:  
- Find the cube of one-digit numbers and the cube root of perfect cubes (less than 1000).  
- Use appropriate tools (e.g., calculator, pencil and paper) to translate large numbers from scientific notation.  
- Identify the y-intercept and calculate the slope of a line from an equation or graph.  
- Graph a system of linear equations and identify the solution as the point of intersection. | The student who just enters Level 3 should be able to:  
- Solve simple quadratic monomial equations and represent the solution as a square root.  
- Work with and perform operations with scientific notation of large numbers.  
- Identify unit rate of change in linear relationships (i.e., slope is the rate of change).  
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms and equations with infinitely many solutions or no solution.  
- Solve a system of linear equations with integer coefficients using an algebraic strategy. | The student who just enters Level 4 should be able to:  
- Write a system of two linear equations with two variables to represent a context. |  

### Algebra ALDs

**RANGE ALD**  
Target: Create equations that describe numbers or relationships.

| Level 1 students should be able to create and use one-step linear equations in one variable to model a familiar situation and to solve a familiar problem. They should be able to graph a linear or a quadratic equation in two variables and be able to rearrange a familiar formula or an unfamiliar linear formula in one or two variables for a particular given quantity. | Level 2 students should be able to create and use quadratic equations, linear equations, and linear inequalities in one and two variables to model a familiar situation and to solve a familiar problem. They should be able to graph a linear or a quadratic equation in two variables and be able to rearrange a familiar formula or an unfamiliar linear formula in one or two variables for a particular given quantity. | Level 3 students should be able to create and use linear, quadratic, and rational equations and inequalities and exponential equations with an integer base and a polynomial exponent in multiple variables to model an unfamiliar situation and to solve an unfamiliar problem. They should be able to graph an equation in two variables and be able to rearrange a linear, a quadratic, an absolute, a rational, or a cubic multi-variable formula for a particular given quantity. | Level 4 students should be able to rearrange polynomial, logarithmic, exponential, or trigonometric formulas with one or more variables to highlight a quantity of interest and be able to analyze in context to determine which quantity is of interest. |

**RANGE ALD**  
Target: Represent and solve equations and inequalities graphically.

| Level 1 students should be able to represent a linear equation with an integer-valued slope in two variables graphically on a coordinate plane. | Level 2 students should be able to represent linear equations and inequalities with integer coefficients in one and two variables graphically on a coordinate plane and should understand that the plotted line or curve represents the solution set to an equation. They should be able to graph and estimate the solution of systems of equations or inequalities. | Level 3 students should be able to represent polynomial, rational, absolute value, exponential, and logarithmic functions graphically. They should be able to graph and estimate the solution of systems of equations and systems of linear inequalities. They should understand that the plotted line, curve, or region represents the solution set to an equation or inequality. | Level 4 students should be able to explain why the x-coordinates of the points where f(x) and g(x) intersect correspond to the solution to f(x) = g(x). |
### Stage 1 Desired Results

#### Transfer Goals

*Students will be able to independently use their learning to...*

- Develop concepts and procedures of representing linear relations to analyze functions.
- Develop and apply procedures of representations of linear relations to model real-world situations.
- Develop procedures of equations/inequalities to solve systems of equations/inequalities.
- Apply procedures of systems of equations/inequalities to real-world applications.

#### Meaning Goals

##### UNDERSTANDINGS

*Students will understand that...*

- Linear functions can be represented by a table, graph, verbal description or equation and that each representation can be transferred to another representation.
- Linear models can be created, used, and interpreted for real-life situations.
- Real world situations can be modeled by systems of linear equations or inequalities.
- A system of equations can have no, one, or infinitely many solutions.
- Solutions of systems of equations are ordered pairs that satisfy all equations.
- Solutions of systems of inequalities are ordered pairs that satisfy all inequalities, often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Multiple methods may be used to solve a system of equations or inequalities.

##### ESSENTIAL QUESTIONS

- What real-world situations can be modeled by a linear relationship?
- How can technology help to determine whether a linear model is appropriate in a given situation?
- How can systems of linear equations or inequalities be used to model real-world situations?
- How can the solution(s) of a system be represented and interpreted?
- What processes may be used to solve a system of equations or inequalities?
- What is/are solutions to a system of equations or inequalities?

#### Acquisition Goals

*Students will know...*  
*Students will be skilled at...*

- When a relationship is a function
- The interpretations of the slope and
- Rearranging formulas to highlight a quantity of interest, using the same reason as in solving equations.
<table>
<thead>
<tr>
<th>Intercepts of a linear function, including equations for parallel and perpendicular lines</th>
<th>Translating among representations of linear functions including tables, graphs, equations and real-life situations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The meaning of solutions to a system of equation</td>
<td>• Determining for what range of values a linear model might be appropriate (restrictions on domain/range) for a given situation.</td>
</tr>
<tr>
<td>• The meaning of solutions to inequalities and systems of inequalities</td>
<td>• Representing constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.</td>
</tr>
<tr>
<td>• The different methods to solve systems of equations</td>
<td>• Relating the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.</td>
</tr>
<tr>
<td>• Domain and range of a function represented in a graph, equation, table, or real-world context</td>
<td>• Defining appropriate quantities for the purpose of descriptive modeling.</td>
</tr>
</tbody>
</table>

- Writing a system of linear equations in two variables to model a situation.
- Determining if an ordered pair is a solution to a system and interpret the viability of solutions.
- Explaining why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions.
- Solving a system of two equations or inequalities graphically, using tables, algebraically or with technology.
- Proving that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- Graphing the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
**Stage 1 Established Goals: Common Core State Standards for Mathematics**

*Students will be able to independently use their learning to...*

**Solve systems of equations**

CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
REI.6 Solve systems of linear equations and approximately (e.g., with graphs) focusing on pairs of linear equations in two variables.

**Represent and solve equations and inequalities graphically.**

REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

**Explanations, Examples, and Comments**

**Vocabulary Cards**
- Solution
- Parallel lines
- Perpendicular lines
- No solution
- Infinitely many solutions
- Systems of equations
- Systems of inequalities
- Boundary line

**Explanations**
Solving systems by graphing, substitution, and elimination was taught in 8th grade. Parallel lines, Perpendicular lines, and systems of inequalities are new to algebra. Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities.

**Stage 3 MATERIALS BY STANDARD(S):**
Teacher should use assessment data to determine which of the materials below best meet student instructional needs. All materials listed may not be needed.

**Extended Algebra:**
Continue working on Linear review

**Book Resources**
- **Holt Algebra 1:** 5-8 Slopes of Parallel and Perpendicular Lines
- **Holt Algebra 1:** 6-1 Solving Systems by Graphing
- **Holt Algebra 1:** 6-2 Solving Systems by Substitution
- **Holt Algebra 1:** 6-3 Solving Systems by Elimination
- **Holt Algebra 1:** 6-4 Solving Special Systems
- **Holt Algebra 1:** 6-6 Solving Systems of Linear
REI.5 Example:
1. Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they two numbers, x and y, satisfy the equations \( x + y = 10 \) and \( x - y = 4 \).

2. The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.

3. Phil and Cath make and sell boomerangs for a school event. They raise money they will go to charity.

They plan to make them in two sizes: small and large.

Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving.

Cath will decorate them. She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity. The large boomerang will make $10 for charity. They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?

REI.6 Examples:

1. José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.

2. Solve the system of equations: \( x + y = 11 \) and \( 3x - y = 5 \). Use a second method to check your answer.

Additional Resources
- Wrestling no guidance
- Wrestling a little guidance
- Wrestling a lot of guidance
- Candle Problem
- Writing Linear Equations: Stations Activity
- Solving 2-by-2 Systems by Graphing: Stations Activity
- Solving Systems of Equations Algebraically (Georgia Task)
- The Tortoise and The Hare
- Solving 2-by-2 Systems by Substitution: Stations Activity
- Rental Car Decisions
- Solving 2-by-2 Systems by Elimination: Stations Activity
- Graphing Inequalities (Georgia Task)
- Solving Systems of Inequalities: Stations Activity
Solve the system of equations: \( x - 2y + 3z = 5, \) \( x + 3z = 11, \) \( 5y - 6z = 9. \) The opera theater contains 1,200 seats, with three different prices. The seats cost $45 per seat, $50 per seat, and $60 per seat. The opera needs to gross $63,750 on seat sales. There are twice as many $60 seats as $45 seats. How many seats in each level need to be sold?

3. A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special.
   - On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials.
   - On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials.

What is the cost of each lunch special?

REI.12 Examples:
1. Graph the solution: \( y < 2x + 3. \)
2. A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women’s magazines, but the company always publishes at least as many women’s magazines as men’s magazines. Find a system of inequalities that describes the possible number of men’s and women’s magazines that the company can produce each year consistent with these policies. Graph the solution set.
3. Graph the system of linear inequalities below and determine if \((3, 2)\) is a solution to the system.
   \[
   \begin{align*}
   x - 3y &> 0 \\
   x + y &\leq 2 \\
   x + 3y &> -3
   \end{align*}
   \]
4. Graph this system of inequalities below on the given coordinate grid.
   \[
   \begin{align*}
   x + y &\geq 12 \\
   20x + 30y &\leq 300
   \end{align*}
   \]

To create a line, click in the grid to create the first point on the line. To create the second point on the line, move the pointer and click. The line will be automatically drawn between the two points. Use the same process to create additional lines.

When both inequalities are graphed, select the region in your graph that represents the solution to this system of inequalities. To select a region, click anywhere in the region. To clear a selected region, click anywhere in the selected region.

5.

Summer Job (Georgia Task)
Schools NYC.gov cycling and fencing
Systems of Equations: TI Activity

Pet Sitters: Develop Understanding Task (A.CED.3) page 4
Too Big or Not Too Big, That is the Questions: Solidify Understanding Task (A.CED.2, A.REI.12) page 7
Some of One, None of the Other: Solidify Understanding Task (A.CED.2, A.CED.4) page 11
Pampering and Feeding Time: Practice Understanding Task (A.CED.2, A.CED.3, A.REI.12) page 15
All for One, One for All: Solidify Understanding Task (A.CED.3, A.REI.12) page 18
Get to the Point: Solidify Understanding Task (A.REI.6) page 23
Shopping for Cats and Dogs: Develop Understanding Task (A.REI.5, A.REI.6) page 26
Can you Get to the Point, Too: Solidify Understanding Task (A.REI.5, A.REI.6) page 29
Food for Fido and Fluffy: Solidify Understanding Task (A.CED.3) page 32
Taken Out of Context: Practice Understanding Task (A.REI.6) page 35
More Things Taken Out of Context: Practice Understanding task (A.REI.12) page 38
The coordinate grid below shows points A through J.

Given the system of inequalities shown below, select all the points that are solutions to this system of inequalities.

\[
\begin{align*}
\{ x + y &< 3 \\
2x - y &> 6
\end{align*}
\]

- ○ A
- ○ B
- ○ C
- ○ D
- ○ E
- ○ F
- ○ G
- ○ H
- ○ I
- ○ J
### Stage 2 - Evidence

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<td>F-IF.8 (M.274)</td>
<td>Performance task</td>
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<td>A-REI.12 (J.012)</td>
<td>Pet Sitters Revisited:</td>
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<td>A-REI.12 (J.087)</td>
<td>Develop Understanding Task</td>
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<td>A-REI.6 (A.032)</td>
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<td>Formative assessments</td>
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<tr>
<td>A.CED.3 and A.REI.5, 6, 12</td>
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<tr>
<td>#1, #2, #3, #4,</td>
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**OTHER ASSESSMENT EVIDENCE:**

1st Semester Summative (sent to building)

### Stage 3 – Learning Plan Sample

*Summary of Key Learning Events and Instruction that serves as a guide to a detailed lesson planning*

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<th>LEARNING ACTIVITIES:</th>
<th>NOTES:</th>
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<td>A suggested guide to layout of unit by days</td>
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</table>

**CED.3, REI.5, REI.6, and REI.12**

Day 1: Parallel Lines
- **Holt Algebra 1**: 5-8 Slopes of Parallel and Perpendicular Lines
- Prentice Hall Algebra 1: 5-6 Parallel and Perpendicular Lines

Day 2: Perpendicular Lines
- **Holt Algebra 1**: 5-8 Slopes of Parallel and Perpendicular Lines
- Prentice Hall Algebra 1: 5-6 Parallel and Perpendicular Lines

Day 3: Systems by Graphing
- Candle Problem and/or
- **Holt Algebra 1**: 6-1 Solving Systems by Graphing

Day 4: Solving Systems by Graphing
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<td>- Ti Systems Activity</td>
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<td>- Some of One, None of the Other</td>
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<td>- Pampering and Feed Time</td>
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<tr>
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<tr>
<td>- Prentice Hall Algebra 1: Solving Systems Using Elimination</td>
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<td>- Shopping for Cats and Dogs</td>
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<td><strong>Day 8: Solving Systems</strong></td>
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<td>- <strong>Holt Algebra 1</strong>: 6-4 Solving Special Systems</td>
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<tr>
<td>- Can You Get to the Point, Too</td>
</tr>
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<td><strong>Day 9: Solving Systems of Linear Inequalities</strong></td>
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<tr>
<td>- <strong>Holt Algebra 1</strong>: 6-6 Solving Systems of Linear Inequalities</td>
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<tr>
<td>- Pet Sitters</td>
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<td><strong>Day 10: Solving Systems of Linear Inequalities</strong></td>
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<td>- Too Big or Not Too Big</td>
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<td><strong>Day 11: Solving Systems of Linear Inequalities</strong></td>
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<td>- Food for Fido and Fluffy</td>
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<td>- Graphing Inequalities (Georgia Task)</td>
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<td><strong>Day 12: Solving Systems of Linear Inequalities</strong></td>
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<td>- Taken Out of Context</td>
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## Stage 3 – Learning Plan Sample

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<tr>
<td>- More Things Taken Out of Context</td>
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<tr>
<td>- Pet Sitters Revisited</td>
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